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Calculus 2

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Preface

This book is a basic source of information for students of mechanical engineering, especially for students of the Faculty of Manufacturing Technologies with a seat in Prešov of the Technical University of Košice. It could also be useful for students of other disciplines if they are interested in the basics of linear algebra, ordinary differential equations, and the differential and integral calculus of functions of several variables.

The book is divided into four chapters. Chapter 1 is focused on the basics of linear algebra, a substantial part is devoted to matrices and systems of linear equations. Chapter 2 is devoted to the basic types of ordinary differential equations and their solutions. Chapter 3 is devoted to the differential calculus of functions of several variables, including applications. Chapter 4 is devoted to double and triple integrals and their geometric and physical applications.

Each chapter of this book contains, in addition to the necessary theory, a number of solved examples as well as exercises. It may therefore be a sufficient source of information for courses that are similar in content to this book.

author

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