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Preface

This book is a basic source of information for students of engineering, especially for students Faculty of Manufacturing Technologies with a seat in Prešov of the Technical University of Košice. It could also be useful for students of other disciplines if they are interested in the calculus of one variable.

The book is divided into 5 chapters. Chapter 1 is devoted to the basic concept of the function of one real variable. One part of it is also devoted to elementary functions. Chapter 2 is devoted to the limit of the function and its continuity. Chapter 3 is devoted to the derivative of a function. Part of it is devoted to applications of derivatives. Chapter 4 is devoted to the indefinite integral. It contains basic concepts as well as methods for calculating many types of indefinite integral. Chapter 5 is devoted to a definite integral and some of its geometric and physical applications.

Each chapter of this book contains, in addition to the necessary theory, a number of solved examples, as well as exercises. It could therefore be a sufficient source of information for completing courses whose content is similar to that of this book.

author
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