

Dušan Knežo  
**Calculus 1**

RAM-Verlag  
2020

This work was realized under the contract of Cultural and Educational Grant Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic KEGA 002TUKE-4/2019.

Title: Calculus 1

Author: © Dušan Knežo, 2020

Reviewers: prof. RNDr. Ján PLAVKA, CSc.  
doc. RNDr. Miriam ANDREJIOVÁ, PhD.

Publisher: RAM-Verlag  
Stüttinghauser Ringstr. 44  
D-58515 Lüdenscheid  
Germany  
RAM-Verlag@t-online.de  
<https://ram-verlag.eu>

The publisher cannot be held responsible for any linguistic errors in the book:  
Such responsibility is only up to the authors.

**ISBN 978-3-96595-000-9**

## Preface

This book is a basic source of information for students of engineering, especially for students Faculty of Manufacturing Technologies with a seat in Prešov of the Technical University of Košice. It could also be useful for students of other disciplines if they are interested in the calculus of one variable.

The book is divided into 5 chapters. Chapter 1 is devoted to the basic concept of the function of one real variable. One part of it is also devoted to elementary functions. Chapter 2 is devoted to the limit of the function and its continuity. Chapter 3 is devoted to the derivative of a function. Part of it is devoted to applications of derivatives. Chapter 4 is devoted to the indefinite integral. It contains basic concepts as well as methods for calculating many types of indefinite integral. Chapter 5 is devoted to a definite integral and some of its geometric and physical applications.

Each chapter of this book contains, in addition to the necessary theory, a number of solved examples, as well as exercises. It could therefore be a sufficient source of information for completing courses whose content is similar to that of this book.

author

# Contents

<b>1</b>	<b>Functions</b>	<b>1</b>
1.1	Function, Domain and Range . . . . .	1
1.2	Properties of Functions . . . . .	3
1.3	Elementary Functions . . . . .	12
1.3.1	Trigonometric Functions . . . . .	17
1.3.2	Cyclometric Functions . . . . .	20
<b>2</b>	<b>Limit and Continuity</b>	<b>25</b>
2.1	Limits . . . . .	25
2.2	Improper Limits . . . . .	25
2.3	One-sided Limits . . . . .	28
2.4	Impottant Formulas . . . . .	31
2.5	Continuity . . . . .	32
2.6	Asymptotes . . . . .	35
<b>3</b>	<b>Derivatives</b>	<b>41</b>
3.1	Derivative . . . . .	41
3.2	Equation of Tangent and Normal Line . . . . .	45
3.3	Higher Order Derivatives . . . . .	47
3.4	L'Hospital Rule . . . . .	49
3.5	Monotonicity of Function . . . . .	50
3.6	Local Extrema . . . . .	52
3.7	Concavity . . . . .	53
3.8	Inflection Point . . . . .	54
3.9	Graph Sketching . . . . .	56
<b>4</b>	<b>Indefinite Integral</b>	<b>79</b>
4.1	Primitive Function and Indefinite Integral . . . . .	79
4.2	Basic Formulas and Rules of Integration . . . . .	80

---

4.3	Integration by Substitution . . . . .	83
4.4	Integration by Parts . . . . .	84
4.5	Integration of Rational Functions . . . . .	86
4.5.1	Rational Functions . . . . .	86
4.5.2	Partial Fractions . . . . .	87
4.5.3	Integration of Partial Fractions . . . . .	89
4.6	Integration of Irrational Functions . . . . .	93
4.7	Integration of Trigonometric Functions . . . . .	101
4.8	Integration of Transcendental Functions . . . . .	106
<b>5</b>	<b>Definite Integral</b>	<b>113</b>
5.1	Concept of Definite Integral . . . . .	113
5.2	Properties of definite integral . . . . .	115
5.3	Newton-Leibniz formula . . . . .	116
5.4	Integration by substitution . . . . .	117
5.5	Integration method by parts . . . . .	118
5.6	Applications of Definite Integral . . . . .	119
5.6.1	Area between Curves . . . . .	119
5.6.2	Volume of Solid of Revolution . . . . .	124
5.6.3	Length of Plane Curve . . . . .	126
5.7	Surface Area of Revolution . . . . .	128
5.7.1	Center of Mass and Moments of Inertia . . . . .	128
	<b>Answers to Exercises</b>	<b>135</b>
	<b>References</b>	<b>139</b>